



Fig. 1. Model structure and relevant variables are shown with respect to the antigenic type ax within a system with two alleles (a and b) at one locus and two alleles (x and y) at another (see model description in text for explanation).

Upper Right) includes only those hosts that have been exposed to types sharing allele a . Fig. 1 *Lower Left* shows exactly the proportion of the host population, Γ_{ax} , that has been subjected to one of the alleles contained in ax but not to both alleles. We assume that these individuals have a lower probability ($1 - \gamma$) of becoming infectious than those who have never been in contact with ax or with any types that share alleles with ax . In Fig. 1 *Lower Right*, we indicate another subset, Δ_{ax} , which contains individuals that have been in contact both with types sharing allele a and with types sharing allele x , but have never been exposed to ax itself. These individuals benefit from an additional reduction in the risk of becoming infectious, ($1 - \delta$), because of the combined exposure to alleles that define ax .

The two key immunological parameters involved are thus, γ and δ . The first, γ , is a measure of the cross-immunity that results from previous exposure to at least one allele contained within an antigenic type. The second parameter, δ , is a measure of the additional cross-immunity arising from accumulated exposure to more than one allele.

We ran this model for various numbers of alleles ($n \times m$) at two loci to determine the occurrence of single strain (antigenic type) epidemics within the (γ, δ) parameter space as indicated by the following measure:

$$\varepsilon = \frac{1}{P} \sum_{i=1}^P \frac{y_{max}^i - y_{sub}^i}{y_{max}^i},$$

where P = number of peaks in some defined time interval, y_{max} = the prevalence of peaking strain, and y_{sub} = the prevalence of the strain with second-highest peak.

Fig. 2 records the measure of single-strain dominance (ε) for various $(n \times m)$ combinations. We find, in line with previous observations (13), that at low levels of cross-immunity, γ , all strains coexist at a common level of prevalence, a state of no strain structure (NSS). This state generates no score on the ε scale and corresponds to the white areas on the left hand side of each Fig. 2 plot. A second steady-state involves the stable dominance of a subset of strains (13), referred to as discrete strain structure (DSS), and occurs in regions where either γ or δ is high (top and right-hand

white areas of each Fig. 2 plot). Separating NSS and DSS is a region of chaotic or cyclical strain structure (CSS) where dominant types are periodically replaced. Within this region, we identified a central core of chaotic behavior that frequently exhibits sharp epidemics dominated by a single strain (see Fig. 3a). For asymmetric $n \times m$ systems (where n is not a multiple of m) the region of single-strain dominance is extended by a number of other behaviors, such as the ordered, alternating appearance of antigenic types of the form $ax \rightarrow by \rightarrow cz \rightarrow aw \rightarrow bx \rightarrow cy$ in a periodic (see Fig. 3b) or quasi-periodic (see Fig. 3c) manner. See [supporting information \(SI\)](#) for further information on the dynamical behavior of the model.

Antigenic Evolution of Influenza. We believe that this model can provide a more comprehensive explanation of influenza dynamics than the prevailing conceptual frameworks described above (2–12). It has been proposed that host immunity structure may play an important role in influenza epidemiology (12) but that such models would still need to incorporate continuous incremental antigenic change. Here, we show that a dynamic network of host immune responses against a small number of functionally constrained epitopes can provide an alternative explanation to this type of model of antigenic drift. Our results demonstrate that single-strain epidemic outbreaks can occur across a broad range of conditions describing the degree of immunological cross-protection conferred by previous exposure (defined by parameters γ and δ). These dynamics are most likely to be observed when cross-protection is not complete, and the additive effects of further exposure are low. That γ is likely to take intermediate values for influenza is indicated by the levels of cross-reactivity observed among different strains (14) as well as reinfection patterns (15). Also, the phenomenon of “original antigenic sin” observed in influenza, whereby current exposure selectively boosts the immune response to an earlier infecting antigenic type (16), would reinforce these dynamics by reducing δ .

The observation that amino acid changes within a set of HA codons are associated with influenza isolates that give rise to epidemics in subsequent years (2, 3) is currently interpreted as support for the prevailing antigenic drift paradigm. However, mutation at these highly evolutionarily constrained sites (see [SI](#)) could as easily lead to the regeneration of antigenic types within an influenza subtype (such as H3N2), which then emerge epidemically as a result of variant-specific cross-protection. Pleiotropic effects of viral mutations (17) are likely to further restrict the variety of possible epitopes within the HA gene, thus justifying its representation as multilocus system with a limited number of alleles at each locus. Crucially, antigenic distances between successive epidemics (which are determined by only a few nucleotide sites) may contract or remain static while the overall genetic divergence of influenza continues to accumulate linearly through time.

And there is indeed empirical evidence that antigenic distances between influenza epidemics do not always increase with time. The antigenic distances among influenza A H3N2 isolates have recently been measured by using hemagglutination inhibition (HI) assays (18). When these data are transformed onto a two-dimensional antigenic space by using multivariate statistics, the H3N2 population displays a zig-zagging, not linear, trajectory (18) (Fig. 4c) whose changes in direction cannot be explained by the current antigenic drift paradigm. Furthermore, comparison with our model suggests that the linear component of this movement is, at least in part, due to use of censored data in the multivariate analysis (18) and not to the sequential accumulation of antigenic distance across epidemics. Specifically, we used a five-locus, two-allele system ($2 \times 2 \times 2 \times 2 \times 2$; Fig. 4a) to compare the antigenic dynamics of our model with influenza A. Mathematically, the antigenic space of our model is a 5-hypercube whose nodes represent the 32 possible antigenic types. However, if our model population is subjected to the same sampling scheme

munogenic, determinants (14, 32) remains the more promising strategy.

Methods

Transmission Dynamics. The two-locus model is governed by the following set of coupled differential equations for each antigenic type ij (defined as allele i at one locus and allele j at the other):

$$\frac{dz_{ij}}{dt} = \lambda_{ij}(1 - z_{ij}) - \mu z_{ij}, \quad \forall i \in \{a, b, c, \dots\} \cap j \in \{x, y, z, \dots\}$$

$$\frac{dw_{ij}}{dt} = \left(\sum_k \lambda_{ik} + \sum_l \lambda_{lj} - \lambda_{ij} \right) (1 - w_{ij}) - \mu w_{ij}$$

$$\frac{dx_i}{dt} = \sum_k \lambda_{ik}(1 - x_i) - \mu x_i$$

$$\frac{dx_j}{dt} = \sum_l \lambda_{lj}(1 - x_j) - \mu x_j$$

$$\frac{dy_{ij}}{dt} = \lambda_{ij}(\Omega_{ij} + (1 - \gamma)\Gamma_{ij} + (1 - \gamma)(1 - \delta)\Delta_{ij}) - \sigma y_{ij}.$$

In the above, z_{ij} is the fraction of the population that has been exposed to pathogen type ij and is now either infected or recovered; y_{ij} is the proportion of the population currently infectious with type ij , consequently, λ_{ij} ($= \beta y_{ij}$) is the force of infection; w_{ij} denotes the proportion of the population that has been exposed to any antigenic type that share alleles with ij (and

includes ij itself), whereas x_i and x_j includes only those that have been exposed to types sharing either allele i or allele j , respectively; the average life expectancy is given by $1/\mu$.

Individuals who have never been in contact with type ij or any types that share alleles with ij , here given as $\Omega_{ij} = 1 - w_{ij}$, have no protection and become infectious. Those individuals that have been exposed to antigenic types sharing alleles at one or the other locus, here denoted as

$$\Gamma_{ij} = \sum_{k=i, j} (w_{ij} - x_k),$$

have a lower probability $(1 - \gamma)$ of becoming infectious; and individuals that have been in contact both with types sharing allele i and types sharing allele j , but have never been exposed to ij itself, denoted as Δ_{ij} ($= w_{ij} - z_{ij} - \Gamma_{ij}$) benefit from an additional reduction in the risk of becoming infectious, $(1 - \delta)$, because of the combined exposure to alleles that define ij .

Antigenic Map. Infected individuals were sampled from our model population at the peaks of successive epidemics in proportion to the relative frequency of their respective antigenic types. Antigenic distances among these sampled individuals were calculated as the number of loci at which they differed. Distances were mapped onto a two-dimensional Euclidean space by using the same multidimensional scaling method developed by Smith *et al.* (18) (available from www.antigenic-cartography.org; best-fit result of 25 runs is shown).

We thank Derek Smith for his critical assistance in generating Fig. 4b and Paul Harvey, Angela McLean, and Jim Kaufman for their extremely helpful comments. We thank the Medical Research Council for financial assistance.

- Dowdle WR, Schild GC (1976) *Bull Pan Am Health Organ* 10:193–195.
- Bush RM, Bender CA, Subbarao K, Cox NJ, Fitch WM (1999) *Science* 286:1921–1925.
- Fitch WM, Bush RM, Bender CA, Cox NJ (1997) *Proc Natl Acad Sci USA* 94:7712–7728.
- Haraguchi Y, Sasaki A (1997) *Phil Trans R Soc London Ser B* 352:11–20.
- Gog JR, Grenfell BT (2002) *Proc Natl Acad Sci USA* 99:17209–17214.
- Gomes MG, Medley GF, Nokes DJ (2002) *Proc R Soc London Ser B* 269:227–233.
- Ferguson NM, Galvani AP, Bush RM (2003) *Nature* 422:428–433.
- Boni MF, Gog JR, Andreasen V, Christiansen FB (2004) *Theor Popul Biol* 65:179–191.
- Finkenstadt BF, Morton A, Rand DA (2005) *Stat Med* 24:3447–3461.
- Koelle K, Cobey S, Grenfell B, Pascual M (2006) *Science* 314:1898–1903.
- Plotkin JB, Dushoff J, Levin SA (2002) *Proc Natl Acad Sci USA* 99:6263–6268.
- Andreasen V, Lin J, Levin SA (1997) *J Math Biol* 35:825–842.
- Gupta S, Ferguson N, Anderson R (1998) *Science* 280:912–915.
- Tumpey TM, Garcia-Sastre A, Taubenberger JK, Palese P, Swayne DE, Basler CF (2004) *Proc Natl Acad Sci USA* 101:3166–3171.
- Davies JR, Grilli EA, Smith AJ (1986) *J Hyg (London)* 96:345–352.
- Webster RG, Kasel JA, Couch RB, Laver WG (1976) *J Infect Dis* 134:48–58.
- Holmes EC (2003) *Trends Microbiol* 11:543–546.
- Smith DJ, Lapedes AS, de Jong JC, Bestebroer TM, Rimmelzwaan GF, Osterhaus AD, Fouchier RA (2004) *Science* 305:371–375.
- Vincent AL, Lager KM, Ma W, Lekcharoensuk P, Gramer MR, Loiacono C, Richt JA (2006) *Vet Microbiol* 118:212–222.
- Chen H, Smith GJ, Li KS, Wang J, Fan XH, Rayner JM, Vijaykrishna D, Zhang JX, Zhang LJ, Guo CT, *et al.* (2006) *Proc Natl Acad Sci USA* 103:2845–2850.
- Smith CB, Cox NJ, Subbarao K, Taber LH, Glezen WP (2002) *J Infect Dis* 185:980–985.
- Kaplan NL, Darden T, Hudson RR (1988) *Genetics* 120:819–829.
- Kaplan NL, Hudson RR, Langley CH (1989) *Genetics* 123:887–899.
- Holmes EC, Ghedin E, Miller N, Taylor J, Bao Y, St. George K, Grenfell BT, Salzberg SL, Fraser CM, Lipman DJ, Taubenberger JK (2005) *PLoS Biol* 3:e300.
- Kaverin NV, Rudneva IA, Ilyushina NA, Varich NL, Lipatov AS, Smirnov YA, Govorkova EA, Gitelman AK, Lvov DK, Webster RG (2002) *J Gen Virol* 83:2497–2505.
- Philpott M, Easterday BC, Hinshaw VS (1989) *J Virol* 63:3453–3458.
- Vong S, Coghlan B, Mardy S, Holl D, Seng H, Ly S, Miller MJ, Buchy P, Froehlich Y, Dufourcq JB, *et al.* (2006) *Emerg Infect Dis* 12:1542–1547.
- Buxton-Bridges C, Katz JM, Seto WH, Chan PK, Tsang D, Ho W, Mak KH, Lim W, Tam JS, Clarke M, *et al.* (2000) *J Infect Dis* 181:344–348.
- Writing Committee WHO Consultation on Human Influenza A/H5 (2005) *N Eng J Med* 353:1374–1385.
- Lipatov AS, Hoffmann E, Salomon R, Yen HL, Webster RG (2006) *J Infect Dis* 194:1040–1043.
- Lu X, Edwards LE, Desheva JA, Nguyen DC, Rekstin A, Stephenson I, Szretter K, Cox NJ, Rudenko LG, Klimov A, Katz JM (2006) *Vaccine* 24:6588–6593.
- Epstein SL, Kong WP, Mispilon JA, Lo CY, Tumpey TM, Xu L, Nabel GJ (2005) *Vaccine* 23:5404–5410.